Bianchi type V non-static barotropic perfect fluid cosmological model in general relativity

RAJ BALI* and RAJNI GUPTA**

*Prof. Raj Bali, Department of Mathematics, University of Rajasthan, Jaipur-302004 (INDIA)
**Ms. Rajni Gupta, Department of Mathematics, University of Rajasthan, Jaipur-302004 (INDIA)
E-mail: balir5@yahoo.co.in

Regd. No. R.N. 52077/90 ISSN 0970-9150

International Journal Of Physical Sciences

(Inter-Disciplinary Journal)

REPRINT

ULTRA SCIENTIST OF PHYSICAL SCIENCES

Post Box - 93

City G.P.O. BHOPAL - 462 001 (India)

Phone : 2533437, 2540624, Mob. 9425014437 Fax No. (00) (91) 755 2710134

e-mail : chiefeditor@ultrascientist.org OR jusps2001@yahoo.com

website : www.ultrascientist.org

Ultra Science Vol. 20(1)M, 231-236 (2008).

Bianchi type V non-static barotropic perfect fluid cosmological model in general relativity

RAJ BALI* and RAJNI GUPTA**

*Prof. Raj Bali, Department of Mathematics, University of Rajasthan, Jaipur-302004 (INDIA).
**Ms. Rajni Gupta, Department of Mathematics, University of Rajasthan, Jaipur-302004 (INDIA)
E-mail: balir5@yahoo.co.in

(Acceptance Date 7th December, 2007)

Abstract

Bianchi Type V non-static barotropic perfect fluid cosmological model in General Relativity, is investigated. To get the deterministic model, we have assumed that (σ^1_1) is proportional to the expantion (θ), where σ^1_1 is the eigen-value of shear tensor (σ^j_i). The stiff fluid model and disordered radiation model are discussed. The physical and geometrical aspects of the model together with singularities in the model are also discussed.

Key words: Bianchi V, barotropic, perfect fluid, non-static.
PACS Nos. 04.20.-q, 98.80.Cq

1. Introduction

Bianchi Type V cosmological models create more interest in the study because of richer structure both geometrically and physically than the standard perfect fluid Friedmann-Robertson-Walker (FRW) models. These models are simple generalization of the negative curvature of FRW models. Heckmann and Schucking[1] have investigated Bianchi Type V cosmological model where matter moves orthogonally to the hypersurface of homogeneity. Exact tilted solutions for Bianchi Type V models are obtained by Ellis[2], Ellis and MacCallum[3], Wainwright *et al.*[4]. Roy and Singh[5] have investigated LRS (Locally Rotationally Symmetric) Bianchi Type V model with viscosity. Banerjee and Sanyal[6] have investigated Bianchi Type V cosmological models with viscosity and heat flow both. Coley[7] has investigated Bianchi Type V imperfect fluid cosmological models which contains both viscosity and heat flow for equation of state $p = (\gamma - 1)\rho$, $0 \leq \gamma \leq 2$. Nayak and Sahoo[9] have obtained Bianchi Type V cosmological models with matter distribution admitting anisotropic pressure and heat flow. Roy and Prasad[8] have investigated some LRS Bianchi Type V cosmological models for heat conduction and radiation. Bali and Sharma[10] have investigated Bianchi type V tilted stiff perfect fluid cosmological model in General Relativity. Bali and Meena[11] have investigated conformally flat tilted Bianchi Type V cosmological model in General Relativity. Bianchi

Type V tilted stiff perfect fluid cosmological model are also investigated by Bali and Meena[12].

In this paper, we have investigated Bianchi Type V non-static barotropic perfect fluid cosmological model in General Relativity. The stiff fluid model and disordered radiation model are also discussed. The physical and geometrical aspects of the model together with singularities in the model are also discussed.

2. *The Metric and Field Equations :*

We consider the Bianchi type V metric in the form

$$ds^2 = -dt^2+A^2dx^2+B^2e^{2x}dy^2+C^2e^{2x}dz^2 \quad (2.1)$$

where A, B and C are functions of t-alone.

Energy momentum tensor T_i^j for perfect fluid is given by

$$T_i^j = (\in +p)\,v_i v^j + p\, g_i^j \quad (2.2)$$

where $\in$ is the energy density, p the isotropic pressure and v^i the flow velocity satisfying

$$g_{ij}v^iv^j = -1$$

We assume that coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3,\ v^4 = 1$$

The Einstein field equation

$$R_i^j - \frac{1}{2}R g_i^j = -8\pi T_i^j \quad (2.3)$$

for the metric (2.1) leads to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}-\frac{1}{A^2}=-8\pi p \quad (2.4)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=-8\pi p \quad (2.5)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-8\pi p \quad (2.6)$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{3}{A^2}=-8\pi \in \quad (2.7)$$

$$\frac{2A_4}{A}+\frac{B_4}{B}-\frac{C_4}{C}=0 \quad (2.8)$$

3. *Solution of Field Equations :*

Equation (2.4) and (2.5) lead to

$$\frac{A_{44}}{A}-\frac{B_{44}}{B}+\left(\frac{A_4}{A}-\frac{B_4}{B}+\right)\frac{C_4}{C}=0 \quad (3.1)$$

From equation (2.5) and (2.6), we have

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}+\right)=0 \quad (3.2)$$

To get the deterministic model, we assume that universe is filled with barotropic perfect fluid distribution. This leads to $p=\gamma\in$ where $0\leq\gamma\leq1$. Now equation (2.4) and equation (2.7) after using barotropic perfect fluid condition ($p=\gamma\in$) lead to

$$\gamma\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+(\gamma+1)\frac{B_4C_4}{BC}+\frac{B_{44}}{B}$$

$$+\frac{C_{44}}{C}=(3\gamma+1)\frac{1}{A^2} \quad (3.3)$$

From equation (2.8), we have

$$A=\ell\,(BC)^{1/2} \quad (3.4)$$

where ℓ is constant of integration.

Equation (3.2) leads to

$$\frac{(CB_4-BC_4)_4}{CB_4-BC_4}=-\frac{A_4}{A} \quad (3.5)$$

On integration, this leads to

$$C^2\left(\frac{B}{C}\right)_4=\frac{m}{A} \quad (3.6)$$

where m is constant of integration.
we assume that

$$BC = \mu,\ B/C = \nu \tag{3.7}$$

Thus, we have

$$B^2 = \mu\nu,\ C^2 = \mu/\nu \tag{3.8}$$

Using equations (3.7) and (3.8) in equation (3.6), we have

$$\frac{\nu_4}{\nu} = \frac{m}{\ell\mu^{3/2}} \tag{3.9}$$

Again using equations (3.4), (3.7) and (3.9) in equation (3.3), we have

$$\frac{\mu_{44}}{\mu} + \frac{3}{4}(\gamma - 1)\frac{\mu_4^2}{\mu^2} + \left(\frac{1-\gamma}{4}\right)\frac{\nu_4^2}{\gamma^2} = \frac{(3\gamma+1)}{\ell^2\mu} \tag{3.10}$$

Using equation (3.9) in equstion (3.10), wehave

$$\mu_{44} + \frac{3}{4}(\gamma - 1)\frac{\mu_4^2}{\mu} + \left(\frac{1-\gamma}{4}\right)\frac{m^2}{\ell^2\mu^2} = \frac{(3\gamma+1)}{\ell^2} \tag{3.11}$$

which leads to

$$f = \left[\frac{4}{\ell^2}\mu + \frac{m^2}{3\ell^2}\frac{1}{\mu} + \frac{M}{\mu^{(3\gamma-1)/2}}\right]^{1/2} \tag{3.12}$$

Where $\mu_4 = f(\mu)$, $\mu_{44} = ff'$, $f' = \frac{df}{d\mu}$ and M is a constant of integration.

To get the deterministic model, we assume two different cases for $\gamma = 1$ (stiff fluid case) and $\gamma = \frac{1}{3}$ (disordered radiation case).

4. *Case (I) (Stiff Fluid Case) :*

Let us assume $\gamma = 1$.
Then equation (3.12) leads to

$$f = \left[\frac{4}{\ell^2}\mu + \frac{m^2}{3\ell^2}\frac{1}{\mu} + \frac{M}{\mu}\right]^{1/2} \tag{4.1}$$

Equation (3.9) and equation (4.1) together leads to

$$\nu = N\left[\frac{1}{\mu} + \frac{1}{\alpha\mu}\sqrt{\mu^2 + \alpha^2}\right]^{-\frac{m}{2\alpha}} \tag{4.2}$$

where

$$\alpha^2 = \frac{m^2 + 3\ell^3 M}{12}$$

Thus, we have

$$A^2 = \ell^2\mu \tag{4.3}$$

$$B^2 = N\mu\left[\frac{1}{\mu} + \frac{1}{\alpha\mu}\sqrt{\mu^2 + \alpha^2}\right]^{-\frac{m}{2\alpha}} \tag{4.4}$$

$$C^2 = \frac{\mu}{N}\left[\frac{1}{\mu} + \frac{1}{\alpha\mu}\sqrt{\mu^2 + \alpha^2}\right]^{\frac{m}{2\alpha}} \tag{4.5}$$

Hence the metric (2.1) leads to

$$ds^2 = -\frac{dt^2}{d\mu^2}d\mu^2 + \ell^2\mu dx^2 + N\mu e^{2x}\left[\frac{1}{\mu} + \frac{1}{\alpha\mu}\sqrt{\mu^2 + \alpha^2}\right]^{-\frac{m}{2\alpha}} dy^2 + \frac{\mu}{N}e^{2x}\left[\frac{1}{\mu} + \frac{1}{\alpha\mu}\sqrt{\mu^2 + \alpha^2}\right]^{\frac{m}{2\alpha}} dz^2 \tag{4.6}$$

Which by suitable transformations leads to

$$ds^2 = \frac{dT^2}{\frac{2}{\ell}\left[T + \frac{\alpha^2}{T}\right]^{1/2}} + \ell^2 T dX^2 + Te^{2X}\left[\frac{1}{T} + \frac{1}{\alpha T}\sqrt{T^2 + \alpha^2}\right]^{-\frac{m}{2\alpha}} dY^2 + Te^{2X}\left[\frac{1}{T} + \frac{1}{\alpha T}\sqrt{T^2 + \alpha^2}\right]^{\frac{m}{2\alpha}} dZ^2 \tag{4.7}$$

where $\mu = T$, $x = X$, $\sqrt{N}y = Y$, $\frac{1}{\sqrt{N}}z = Z$

Physical and Geometrical Features :

The energy density ($\in$), the isotropic pressure (p) for model (4.7) are given by

$$8\pi \in = \frac{3M}{4T^3} \tag{4.8}$$

$$8\pi p = \frac{3M}{4T^3} \tag{4.9}$$

The expansion (θ) and components of shear tensor (σ_i^j) are given by

$$\theta = \frac{3}{\ell T^{3/2}}[T^2 + \alpha^2]^{1/2} \tag{4.10}$$

$$\sigma_1^1 = 0 \tag{4.11}$$

$$\sigma_2^2 = \frac{m}{2\ell T^{3/2}} \tag{4.12}$$

$$\sigma_3^3 = \frac{m}{2\ell T^{3/2}} \tag{4.13}$$

$$\sigma_4^4 = 0 \tag{4.14}$$

Thus

$$\sigma^2 = \frac{1}{2}\left[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2\right]$$

Therefore

$$\sigma = \frac{m}{2\ell T^{3/2}} \tag{4.15}$$

The spatial volume (R^3) and deceleration parameter (q) are given by

$$R^3 = \ell T^{3/2} \tag{4.16}$$

$$q = \frac{2\alpha}{T^2\left(1 + \frac{\alpha^2}{T^2}\right)} \tag{4.17}$$

Discussion

The reality conditions (i) $\in + p > 0$, (ii) $\in + 3p > 0$ given by Ellis[13] lead to $M > 0$.

The model (4.7) starts with a big-bang at $T = 0$ and the expansion in the model decreases as time increases. Since $\frac{\sigma}{\theta} \neq 0$. Hence it represents an anisotropic universe in general. However, for large value T, $\sigma = 0$. Hence the model isotropizes for large values of T. The spatial volume increases as time increases. Since $q > 0$ as $\sigma > 0$, therefore the expansion in the model decreases continuously and for large value of T, it dies out i.e. it represents a decelerating universe.

5. *Case (II) (Disorder radiation case):*

We assume $\gamma = 1/3$.
Equation (3.12) leads to

$$f = \left[\frac{4}{\ell^2}\mu + \frac{m^2}{3\ell^2}\frac{1}{\mu} + M\right]^{1/2} \tag{5.1}$$

To get deterministic value of μ, we assume $m = 0$.
On integration, equation (5.1) leads to

$$\mu = \left(\frac{t}{\ell} + \beta\right)^2 - \frac{M\ell^2}{4} \tag{5.2}$$

where β is constant of integration.
Using our assumption $m = 0$, equation (3.9) leads to

$$\frac{\nu_4}{\nu} = 0$$

which gives

$$\nu = \text{constant} = L \text{ (say)}$$

Thus, we have

$$A^2 = \ell^2\mu = \ell^2\left[\left(\frac{t}{\ell}+\beta\right)^2 - \frac{M\ell^2}{4}\right] \tag{5.3}$$

$$B^2 = \mu\nu = L\left[\left(\frac{t}{\ell}+\beta\right)^2 - \frac{M\ell^2}{4}\right] \tag{5.4}$$

$$C^2 = \frac{\mu}{\nu} = \frac{1}{L}\left[\left(\frac{t}{\ell}+\beta\right)^2 - \frac{M\ell^2}{4}\right] \tag{5.5}$$

Hence the metric (2.1) leads to

$$ds^2 = -dt^2 + \left[\left(\frac{t}{\ell}+\beta\right)^2 - \frac{M\ell^2}{4}\right]\left[\ell^2 dx^2 + Le^{2x}dy^2 + \frac{1}{L}e^{2x}dz^2\right] \tag{5.6}$$

After using the transformations

$$\ell x = X, \quad \sqrt{L}y = Y, \quad \frac{1}{\sqrt{L}}z = Z, \quad \frac{t}{\ell}+\beta = T,$$

the metric (5.6) leads to

$$ds^2 = -\ell^2 dT^2 + \left(T^2 - \frac{M\ell^2}{4}\right)\left(dX^2 + e^{\frac{2X}{\ell}}dY^2 + e^{\frac{2X}{\ell}}dZ^2\right) \tag{5.7}$$

Physical and Geometrical Features :

The energy density ($\in$) and the isotropic pressure (p) for the model (5.7) are given by

$$8\pi \in = \frac{3M}{4\left(T^2 - \frac{M\ell^2}{4}\right)^2} \tag{5.8}$$

$$8\pi p = \frac{M}{4\left(T^2 - \frac{M\ell^2}{4}\right)^2} \tag{5.9}$$

The expansion (θ) and components of shear tensor (σ_i^j) are given by

$$\theta = \frac{3}{2}\left[\frac{4}{\ell} + \frac{M}{T^2} - \frac{M\ell^2}{4}\right] \tag{5.10}$$

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right] = 0 \tag{5.11}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right] = 0 \tag{5.12}$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right] = 0 \tag{5.13}$$

$$\sigma_4^4 = 0 \tag{5.14}$$

Thus we have

$$\sigma = 0 \tag{5.15}$$

The spatial volume (R^3) and the deceleration parameter (q) are given by

$$R^3 = \ell\left(T^2 - \frac{M\ell^2}{4}\right)^{3/2} \tag{5.16}$$

$$q = \frac{M\ell^2}{4T^2} \tag{5.17}$$

Discussion

The reality conditions (i) $\in + p > 0$ and (ii) $\in + 3p > 0$ given by Ellis[13] lead to M>0. The model (5.7) starts with a big-bang at $T = \frac{\ell\sqrt{M}}{2}$. When $T > \frac{\ell\sqrt{M}}{2}$ then the expansion in the model decreases as time increases. The model (5.7) represents a realistic and isotropic universe. The spatial volume (R^3) increases when $T > \frac{\ell\sqrt{M}}{2}$. Since the deceleration parameter q>0, hence the expansion in the model continuously decreases and it represents a decelerating universe. There is a Point Type singularity in the model at

$$T = \frac{\ell\sqrt{M}}{2} \text{ (MacCallum[14])}$$

References

1. Heckmann, O. and Schuking, E., In Gravitation, An Introduction to Current Research ed. Witteo, L., John Wiley, New York (1962).
2. Ellis, G.F.R., *J. Math. Phys., 8,* 1171 (1967).
3. Ellis, G. F. R. and MacCallum, M. A. H., *Comm. Math. Phys., 12,* 108 (1969).
4. Wainwright, J., Ince, W.C.W. and Marshman, B.J., *Gen. Rel. Grav., 10,* 259 (1979).
5. Roy, S.R. and Singh, J.P., *Astrophys. And Space-Science, 96,* 303 (1983).
6. Banerjee, A. and Sanyal, A.K., *Gen. Rel. Grav., 20,* 103 (1988).
7. Coley, A.A., Gen. Rel. Grav., 22, 3 (1990).
8. Nayak, B.K. and Sahoo, B.K., *Gen. Rel. Grav., 28,* 251 (1996).
9. Roy, S.R. and Prasad, A., *Gen. Rel. Grav., 26,* 939 (1994).
10. Bali, Raj and Sharma, K., Progress of Mathematics, B.H.U. (India), 37, 53 (2003).
11. Bali, Raj and Meena, B.L., Pramana - *J. Phys., 62,* 5 (2004).
12. Bali, Raj and Meena, B. L., Proc. Of National Acad. Of Sciences (India), 75A IV (2005).
13. Ellis, G.F.R., General Relativity and Cosmology ed. R.K. Sachs, Clarendon Press, Oxford, p.l 17 (1971).
14. MacCallum, M.A.H., *Comm. Math. Phys. 20,* 57 (1971).